

Quantum Field Theory and Dense Measurement

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We show, using quantum field theory (QFT), that performing a large number of identical repetitions of the same measurement does not only preserve the initial state of the wave function (the Zeno effect), but also produces additional physical effects. We first discuss the Zeno effect in the framework of QFT, that is, as a quantum field phenomenon. We then derive it from QFT for the general case in which the initial and final states are different. We use perturbation theory and Feynman diagrams and refer to the measurement act as an external constraint upon the system that corresponds to the perturbative diagram that denotes this constraint. The basic physical entities dealt with in this work are not the conventional once-performed physical processes, but their n times repetition where n tends to infinity. We show that the presence of these repetitions entails the presence of additional excited state energies, and the absence of them entails the absence of these excited energies.

KEY WORDS: quantum field theory; Zeno effect; Feynman diagrams.

1. INTRODUCTION

The problem of obtaining additional physical effects only because of multiple repetitions of the same measurement or interaction has been discussed both analytically (Bixon, 1982; Giulini *et al.*, 1996; Harris and Stodolsky, 1981; Misra and Sudarshan, 1977; Pascazio and Namiki, 1994; Peres, 1989, Peres and Ron, 1990; Simonius, 1978), and experimentally (Cook, 1988; Itano *et al.*, 1990; Kofman and Kurizki 1996; Kurizki *et al.*, 1995; Wilkinson *et al.*, 1997). These phenomena, in which one may preserve in time an initially prepared state or even “guide” its time evolution to another final predetermined state (Aharonov and Vardi, 1980; Facchi *et al.*, 1999), in contrast to the known rules of quantum mechanics by which the result of measurement cannot be known beforehand (Merzbacher, 1961), are collectively termed quantum Zeno effect (Bixon, 1982; Cook, 1988; Giulini *et al.*, 1996; Harris and Stodolsky, 1981; Itano *et al.*, 1990; Kofman and Kurizki, 1996; Kurizki *et al.*, 1995; Misra and Sudarshan, 1977; Pascazio and Namiki, 1994; Peres, 1989; Peres and Ron, 1990; Simonius, 1978). They were discussed exclusively at

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the level of either the Schroedinger equation (Aharonov and Vardi, 1980; Giulini *et al.*, 1996; Misra and Sudarshan, 1977), or by using the density matrix (Bixon, 1982; Harris and Stodolsky, 1981; Simonius, 1978). In this work we describe these phenomena, using specific examples, in the context of quantum field theory. Moreover, it has been shown (Aharonov and Vardi, 1980), using the spin example, that these repetitions not only preserve or guide to some predetermined state but also may result in entirely new effects as will be explained. We show, using quantum field theory, that this is indeed the case and not only in the spin case. We show this for the two most discussed cases in relation to the many body problem in quantum field theory (Mahan, 1993; Mattuck, 1976): (1) The many body system in which the constituent particles are not interacting with one another, but are submitted to an external potential V , and (2) The many body system in which the constituent particles are interacting with one another. In both cases the single particle propagator can be represented by an infinite series from which we can get the energies and the lifetime of the relevant system (Mahan, 1993; Mattuck, 1976). By “single particle propagator” we mean especially the specific Green function $iG^+(k_2, k_1, t_2 - t_1)_{t_2 > t_1}$ which is the probability amplitude that if at the time t_1 we add a particle in state $\theta_{k_1}(r)$ to the system in its ground state, then at the time t_2 the system will be found in its ground state with an added particle in the state $\theta_{k_2}(r)$ (Mattuck, 1976). The propagator $iG^+(k_2, k_1, t_2 - t_1)_{t_2 > t_1}$ is termed the “dressed” or “clothed” propagator to differentiate it from the free (bare) propagator $iG_0^+(k_2, k_1, t_2 - t_1)_{t_2 > t_1}$ which has the same meaning of a probability amplitude as that of $iG^+(k_2, k_1, t_2 - t_1)_{t_2 > t_1}$, but with no perturbing interaction resulting from either an external potential or from some interaction among the particles composing the system.

We remark that the “clothed” propagator is conventionally estimated (Mattuck, 1976), (Mahan, 1993) by summing to an infinite order over some selective series which is always characterized by the same basic diagram (from a very large number of possible diagrams) repeated to all orders. From the sum over this series one derives physical results like the ground and excited energy states of the system (Mahan, 1993; Mattuck, 1976). That is, the physical phenomena appear after summing to infinite order over this set of series of repetitions of the same diagram. There exists a large number of examples corroborating this. The known Hartree (Mahan, 1993; Mattuck, 1976) and Hartree-Fock (Mattuck, 1976), (Mahan, 1993) quantum field realizations of physical phenomena are the results of summing to an infinite order over only the same repeated diagram. That is, over only the bubble terms (Mahan, 1993; Mattuck, 1976) in the first case, and over only the bubble and open oyster terms in the second case (Mattuck, 1976). Likewise, the random phase approximation method (RPA) is based upon summing over only the terms called the ring terms (Mattuck, 1976). The basic phonon relations are derived (Mattuck, 1964, 1976) from summing to an infinite order over only the same repeated (to all orders) process which represents the Einstein constant frequency

phonon. The plasmon characteristics have been derived by summing over only the “pair bubbles” terms (Mattuck, 1976). Even the two particle propagator is handled by summing over only what is termed the ladder terms (Mattuck, 1976). For all the above and many other cases this summing over the same repeated process results in a new particle, the quasiparticle (Mattuck, 1976), with a characteristic energy, an effective mass, and a finite lifetime. These infinite repetitions over the same process dress the initial “bare” particle and transform it to another one with different energy, mass, and lifetime. We will show in Section 3 that if we have no repetitions then we have also no quasiparticles and no excited energy states.

In the following we refer to the measurement process as a constraint that is externally imposed upon the system so that the experiment corresponds to the Feynman diagram that signifies this constraint. In such a way we may use the formalism of the perturbative quantum field theory, and especially the summing to all orders process, for discussing these experiments and their repetitions. Each order of the perturbative sum, for example order 3 of the Feynman diagram that denotes the bubble process, corresponds to performing three times the analogous experiment that reproduce this process in the same time interval. Thus, the higher order terms of the perturbative series correspond to repeating this experiment a large number of times in the same given time interval which is the Zeno process. That is, we may regard the higher order terms of the selective series of perturbative QFT as corresponding to the Zeno processes of the experiment that reproduces the interaction that is symbolized by the Feynman diagram related to this series. We note that the same equivalence has enabled Aharonov and Vardi (1980) to assign physical meaning to the Feynman paths.

Thus, according to the previous discussion, the starting point will not be the general series which is not summable (Mahan, 1993; Mattuck, 1976), but a selective series which is generally a series of only one process (from actually a very large number of possible processes) and all its different orders. Here, in order to emphasize this element of repetition and its essential role in the formation of the Zeno effect (Bixon, 1982; Giulini *et al.*, 1996; Harris and Stodolsky, 1981; Misra and Sudarshan, 1977; Pascazio and Namiki, 1994; Peres, 1989; Peres and Ron, 1990; Simonius, 1978), we discuss a special version of the last series in which the terms of these series are not all the orders of the once performed relevant interaction, but *all the orders of the n times repetitions of it*, as will be explained in the following sections. Also, using the bubble and open-oyster examples we illustrate the Aharonov–Vardi conclusion (Aharonov and Vardi, 1980), with respect to spin rotations, that even when the physical mechanisms (potentials and interactions), that cause the time evolutions of the physical systems, are absent, nevertheless, the large number of repetitions of the “measurement” of the corresponding observables induces this type of time evolution. In our case we obtain, by these repetitions, an induced continuous spectrum of excited state energies in a finite interval.

In Section 2 use is made of the vacuum amplitude $R(t)$ (Mahan, 1993; Mattuck, 1976) and the unique nature of the Zeno effect (Aharonov and Vardi, 1980; Simonius, 1978) to show this effect for the bubble process (Mahan, 1993; Mattuck, 1976), and for the general unlinked diagram with n identical links (Mattuck, 1976). In Section 3 the Zeno effect is shown also for the case in which the initial and final states of the system are different. This is demonstrated for the specific open-oyster process (Mattuck, 1976), and for the general case of different initial and final states of the system in which the amplitude has a value greater than unity.

2. THE ZENO EFFECT OF THE BUBBLE PROCESS

The vacuum amplitude, as defined in the literature (see, for example, (Mahan, 1993; Mattuck, 1976)), takes into account all the various processes that lead from the ground state, back to the same state. Here, in order to discuss the Zeno effect (Aharonov and Vardi, 1980; Itano *et al.*, 1990; Misra and Sudarshan, 1977) which is characterized by a large number of repetitions of the same process, we adopt a restricted vacuum amplitude formalism that involves repetitions of only one particular process. As we have pointed out, the Hartree and Hartree-Fock procedures, for example, belong to this category.

As mentioned, our basic diagram is the n times repetitions of the process that begins and ends at the same state, where in the limit of dense measurement n tends to be a very large number. That is, this basic diagram is, actually, composed of n identical parts. Thus, the terms of the infinite series representing the vacuum amplitude must signify the different orders of this basic n -times-repeated interaction. The first term of this infinite series is the free term when no interaction occurs in the time interval $(t - t_0)$ (we specify the initial time by t_0). The value of this first term of the vacuum amplitude is unity (Mattuck, 1976), since it expresses the fact that in the unperturbed case the probability amplitude for the quantum system to stay in its ground state is unity. The second term denotes the basic diagram, just described. The third term denotes the probability when this n -times-repeated interaction is performed twice in the time interval $(t - t_0)$ etc. As an example for this process we take the bubble interaction (Haken, 1981; Mahan, 1993; Mattuck, 1976) in which an external potential lifts the system at the time t out of its initial state l creating a hole, and instantaneously puts it back in, destroying the hole. In the energy-time representation the probability amplitude for the occurrence of the bubble process is given by (Mahan, 1993; Mattuck, 1976).

$$L_{\text{bubble}}(l, t) = -i \int_{t_0}^t V_{ll} G^-(l, t_1 - t_1) dt_1, \quad (1)$$

where V_{ll} is the external potential that transmits the system from the state l back again to the same state l . V_{ll} does not depend on t so it can be moved out of

the integral sign in Eq. (1). The point correlation function $iG^-(l, t_1 - t_1)$ is the probability amplitude that at the time t_1 a hole in state l has been added and instantaneously removed (destroyed) from the system in its ground state (Mahan, 1993; Mattuck, 1976). The value of $iG^-(l, t_1 - t_1)$ is -1 (see Mattuck, 1976). The minus sign in Eq. (1) is for the fermion loop (Mattuck, 1976) of the bubble process. The integration time from t_0 to t is the time it takes this process to occur. If this bubble interaction is repeated n times over the same total finite time $(t - t_0)$, we obtain for the probability amplitude to find the system at time t to have the same state it has at time t_0 (Haken, 1981; Mattuck, 1976)

$$\begin{aligned}
 L_{\text{bubble}}^n(l, t) &= (-i)^n \int_{t_0}^t V_{ll} G^-(l, t_1 - t_1) dt_1 \int_{t_0}^{t_1} V_{ll} G^-(l, t_2 - t_2) dt_2 \\
 &\dots \int_{t_0}^{t_{n-1}} V_{ll} G^-(l, t_n - t_n) dt_n = (-i)^n \frac{1}{n!} \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \\
 &\dots \int_{t_0}^t dt_n T_D \underbrace{[G^- \dots G^-]}_n V_{ll}^n \tag{2}
 \end{aligned}$$

where T_D is the Dyson time ordered product operator (Enz, 1992; Mahan, 1993; Mattuck, 1976). The division by $n!$ is because we take into account all the possible orders of the times $t_1, t_2, t_3 \dots t_n$. Here each iG^- has the same constant value (-1 as we have remarked), so we obtain from the equation (2)

$$L_{\text{bubble}}^n(l, t) = \frac{1}{n!} \left(\int_{t_0}^t dt (-iG^-) V_{ll} \right)^n \tag{3}$$

The last equation is the probability amplitude to find the system at the time t , after it has been interacted upon n times by the same bubble interaction, to have the same state it has at the time t_0 . Now, as we have mentioned we must take into account all the possible orders of this n times repeated interaction. If, for example, this n th order interaction is repeated two, three, and four times over the same finite total time $(t - t_0)$, we obtain for the relevant probability amplitudes $(\frac{1}{2!}) (\frac{1}{n!} (\int_{t_0}^t dt (-iG^-) V_{ll})^n)^2$, $(\frac{1}{3!}) (\frac{1}{n!} (\int_{t_0}^t dt (-iG^-) V_{ll})^n)^3$, and $(\frac{1}{4!}) (\frac{1}{n!} (\int_{t_0}^t dt (-iG^-) V_{ll})^n)^4$ respectively. The divisions by $2!$, $3!$, and $4!$ take into account the possible time orders among these n th order interactions (repeated two, three, and four times) besides the extra $n!$ times permutations for each such n times repeated interaction. We note that since, as we have remarked, each such n th order interaction is treated as *the basic interaction* its n parts are not time permuted with the n parts of any other identical basic interaction. Repeating this n th order bubble process n times, and taking the former equations into account we obtain for the probability amplitude

(denoted by P) to find the system in the time t to be in the same state it was in the initial time t_0 .

$$\begin{aligned}
 P_{\text{bubble}}^n(l, t) &= 1 + \frac{1}{n!} \left(\int_{t_0}^{t_n} dt (-iG^-) V_{ll} \right)^n + \frac{1}{2!} \left(\frac{1}{n!} \left(\int_{t_0}^{t_n} dt (-iG^-) V_{ll} \right)^n \right)^2 \\
 &\quad + \frac{1}{3!} \left(\frac{1}{n!} \left(\int_{t_0}^{t_n} dt (-iG^-) V_{ll} \right)^n \right)^3 + \dots \\
 &\quad + \frac{1}{n!} \left(\frac{1}{n!} \left(\int_{t_0}^{t_n} dt (-iG^-) V_{ll} \right)^n \right)^n = 1 + \frac{L_{\text{bubble}}^n}{n!} \\
 &\quad + \frac{1}{2!} \left(\frac{L_{\text{bubble}}^n}{n!} \right)^2 + \frac{1}{3!} \left(\frac{L_{\text{bubble}}^n}{n!} \right)^3 + \dots + \frac{1}{n!} \left(\frac{L_{\text{bubble}}^n}{n!} \right)^n \quad (4)
 \end{aligned}$$

We are interested in showing the existence of the Zeno effect in the limit of dense measurement, that is, of a very large n . We obtain

$$\begin{aligned}
 \lim_{n \rightarrow \infty} P_{\text{bubble}}^n(l, t) &= \lim_{n \rightarrow \infty} \left(1 + \frac{L_{\text{bubble}}^n}{n!} + \frac{1}{2!} \left(\frac{L_{\text{bubble}}^n}{n!} \right)^2 + \frac{1}{3!} \left(\frac{L_{\text{bubble}}^n}{n!} \right)^3 + \dots \right) \\
 &= \lim_{n \rightarrow \infty} \exp \left(\frac{L_{\text{bubble}}^n}{n!} \right) = 1 \quad (5)
 \end{aligned}$$

That is, the probability to remain with the initial state after all these interactions is unity which is the Zeno effect (Aharonov and Vardi, 1980; Giulini *et al.*, 1996; Harris and Stodolsky, 1981; Itano *et al.*, 1990; Misra and Sudarshan, 1977; Peres and Ron, 1990). We can generalize from the specific bubble interaction to a general one. The only condition this general interaction has to fulfil is to start and end at the same state, so that when it is repeated n times, the resulting n th order diagram is composed of n unlinked identical links. Now, it is known (Mahan, 1993; Mattuck, 1976) that the value of an unlinked diagram with n unlinked links L is $\frac{L^n}{n!}$, no matter what is the character of L . Thus, denoting our fundamental generalized interaction by L , and repeating the same process, as we have done for the bubble interaction, we obtain the following vacuum probability amplitude P_{zeno} (to start and end at the same state) in the Zeno limit

$$\begin{aligned}
 \lim_{n \rightarrow \infty} P_{\text{zeno}}(t) &= \lim_{n \rightarrow \infty} \left(1 + \frac{L^n}{n!} + \frac{1}{2!} \left(\frac{L^n}{n!} \right)^2 + \frac{1}{3!} \left(\frac{L^n}{n!} \right)^3 \right. \\
 &\quad \left. \dots + \frac{1}{n!} \left(\frac{L^n}{n!} \right)^n + \dots \right) = \lim_{n \rightarrow \infty} \exp \left(\frac{L^n}{n!} \right) = 1 \quad (6)
 \end{aligned}$$

That is, the quantum Zeno effect may occur in the framework of quantum field theory. This derivation is general in that we do not have to specify the fundamental repeated interaction L .

The same conclusion can also be obtained by considering the ground state energy of the perturbed system which is obtained by using the vacuum amplitude from Eq. (6). This ground state energy is obtained from the following relation, known as the linked cluster theorem (Mattuck, 1976)

$$E_0 = W_0 + \lim_{t \rightarrow \infty(1-i\eta)} i \frac{d}{dt} (\ln R(t)), \tag{7}$$

where W_0 is the ground state energy of the unperturbed Hamiltonian corresponding to the unperturbed ground state θ_0 which is assumed to be the initial state of the system, and η is a positive infinitesimal such that $\eta \cdot \infty = \infty$, and $\eta \cdot C = 0$ for any finite C . The limit $t \rightarrow \infty(1 - i\eta)$ means that first the value of t is set to $T(1 - i\eta)$ and then we take $T \rightarrow \infty$ (see Appendix C in Mattuck, 1976). $R(t)$, in our case, is the $P_{\text{Zeno}}(t)$ from Eq. (6). One sees from the general linked cluster expansion given, for example, by Mattuck (1976, p. 110) that the expansion (6) results from including only the bubble contribution. Thus, substituting in Eq. (7) for $R(t)$ ($P_{\text{Zeno}}(t)$ from Eq. (6)) we obtain (Mattuck, 1976)

$$E_0 = W_0 + \lim_{t \rightarrow \infty(1-i\eta)} i \frac{d}{dt} \left(\ln \left(\lim_{n \rightarrow \infty} e^{\frac{L^n}{n!}} \right) \right) = W_0 + \lim_{t \rightarrow \infty(1-i\eta)} i \frac{d}{dt} (\ln(1)) = W_0 \tag{8}$$

Thus, we see that in the Zeno limit the initial energy (the initial state) is preserved. This is true for any general process L , such that when repeated n times the value of its n unlinked parts diagram (we are restricted here to the vacuum amplitude case) is $\frac{L^n}{n!}$. All we have to do is to use the general $P_{\text{Zeno}}(t)$ from Eq. (6), and Eq. (7). The result we obtain is identical to Eq. (8).

All our discussion thus far of the bubble Zeno effect uses the vacuum amplitude, and so is restricted to the case where the initial and final states of the system were the ground state. We generalize now to any other state and take into account explicitly the unperturbed propagators which connect neighbouring interactions. Here also our basic unit is, because of the Zeno effect, the n -times-repeated bubble interaction. This general bubble process is now more natural than the former, since each bubble interaction is naturally related to the former and to the following identical ones by connecting paths which are the free propagators $G_0^+(l, t_2 - t_1)$ defined as the free propagation of the system from the time t_1 to t_2 without any disturbance whatever. Thus, in order to accomodate to this situation we have to multiply each fundamental bubble process given by Eq. (1) by the free propagators $G_0^+(k, t_1 - t_0)$ and $G_0^+(k, t_2 - t_1)$, the first leads from the initial time t_0 to the time of the interaction t_1 and the second from t_1 to the time after the interaction t_2 , so that Eq. (1) would be written as

$$L_{\text{bubble}}(k, t) = -i \int_{t_0}^t V_{klkl} G_0^+(k, t_1 - t_0) G_0^+(k, t_2 - t_1) G^-(l, t_1 - t_1) dt_1, \tag{9}$$

where k is the initial and final state of each such fundamental bubble process. The interaction is denoted now by V_{klkl} that signifies that our system begins and ends

at the same state k , creating and destroying a hole in state l (if the system interacts only with an external potential then this interaction is denoted by V_{kk} as is done for the vacuum amplitude case). V_{klkl} is a probability amplitude that does not depend on time and is given by (Mattuck, 1976)

$$V_{klkl} = \int d^3\mathbf{r}\phi_k^*(\mathbf{r}) \int |\phi_l(\mathbf{r})|^2 V(\mathbf{r} - \mathbf{r}) d^3\mathbf{r}\phi_k(r),$$

and G^- has the same meaning as in the former case. The free propagator $G_0^+(k, t_2 - t_1)$ has the following value (Mahan, 1993; Mattuck, 1976)

$$G_0^+(k, t_2 - t_1) = \begin{cases} -i\Theta_{t_2-t_1} e^{-i\epsilon_k(t_2-t_1)} & \text{for } t_2 \neq t_1 \\ 0 & \text{for } t_2 = t_1 \end{cases} \quad (10)$$

with

$$\Theta_{t_2-t_1} = \begin{cases} 1 & \text{if } t_2 > t_1 \\ 0 & \text{if } t_2 \leq t_1 \end{cases}$$

Substituting from Eq. (10) into Eq. (9) we obtain

$$L_{\text{bubble}}(k, t) = i \int_{t_0}^t V_{klkl} e^{-i\epsilon_k(t_1-t_0)} e^{-i\epsilon_k(t_2-t_1)} G^-(l, t_1 - t_1) dt_1 \quad (11)$$

Now, since we deal with identical repetitions of the same interaction all the V_{klkl} 's are the same. Also all the ϵ_k 's are, for the same reason, identical to each other. Moreover, we can take also the time differences $(t_n - t_{n-1})$, especially for large n , to be the same. Thus, taking these considerations into account, we write the relevant modified form of Eq. (2) as follows

$$\begin{aligned} L_{\text{bubble}}^n(k, t) &= (-i)^n \int_{t_0}^t \int_{t_0}^{t_1} \dots \int_{t_0}^{t_{n-1}} \underbrace{V_{klkl} \dots V_{klkl}}_n G^-(l, t_1 - t_1) G^-(l, t_2 - t_2) \\ &\quad \dots G^-(l, t_n - t_n) \cdot e^{-i\epsilon_k(t_1-t_0)} e^{-i\epsilon_k(t_2-t_1)} \dots e^{-i\epsilon_k(t_n-t_{n-1})} dt_1 dt_2 \dots dt_n \\ &= (-i)^n (V_{klkl})^n \underbrace{[G^- \dots G^-]}_n \int_{t_0}^t \int_{t_0}^{t_1} \dots \int_{t_0}^{t_{n-1}} \cdot e^{-i\epsilon_k(t_n-t_0)} dt_1 dt_2 \dots dt_n \\ &= (V_{klkl})^n \int_{t_0}^t \int_{t_0}^{t_1} \dots \int_{t_0}^{t_{n-2}} \left(\frac{e^{-i\epsilon_k(t_{n-1}-t_0)}}{-i\epsilon_k} - \frac{1}{(-i\epsilon_k)} \right) dt_1 dt_2 \dots dt_{n-1} \\ &= (V_{klkl})^n \left(\frac{e^{-i\epsilon_k(t-t_0)}}{(-i\epsilon_k)^{n-1}} - \frac{1}{(-i\epsilon_k)^{n-1}} - \frac{(t-t_0)}{(-i\epsilon_k)^{n-2}} - \frac{(t-t_0)^2}{(-i\epsilon_k)^{n-3}} - \dots \right) \\ &= (V_{klkl})^n \left(\frac{e^{-i\epsilon_k(t-t_0)}}{(-i\epsilon_k)^{n-1}} - \sum_{m=0}^{n-1} \frac{(t-t_0)^m}{m!(-i\epsilon_k)^{n-1-m}} \right) \end{aligned} \quad (12)$$

Here, we have taken (Mattuck, 1976) $(-iG^-) = 1$. Expanding the exponent $e^{-i\epsilon_k(t-t_0)}$ in a Taylor series we obtain from the last equation

$$L^n_{\text{bubble}}(k, t) = (V_{klkl})^n \sum_{m=n}^{+\infty} \frac{(t - t_0)^m}{m!(-i\epsilon_k)^{n-1-m}} \tag{13}$$

The left hand side of Fig. 1 shows the n times repetitions of the bubble process which is represented as a circle. These unconnected repetitions conform to Eq. (2). The right hand side of the figure shows these n times repetitions connected by leading paths, and so they conform to Eq. (13).

We note that since what interests us in this work is the limit of very large n of these n -times repeated interactions, represented by equations (12)–(13) in this section and Eq. (28) in the following one, these n multiple interactions are to be regarded as one connected unseparated process (see the discussion before Eq. (4)) and not as repetitions over improper self energy parts (Mattuck, 1976),² so we can use the following Dyson’s equation (Mahan, 1993; Mattuck, 1976) as we have done in equations (18), (29), and (34).

$$\int_{t_0}^t dt_1 \dots \int_{t_0}^{t_{n-1}} dt_n H_1(t_1) \dots H_1(t_n) = \frac{1}{n!} \int_{t_0}^t dt_1 \dots \int_{t_0}^t dt_n T_D[H_1(t_1) \dots H_1(t_n)], \tag{14}$$

where T_D is the Dyson’s time ordered product. The right hand side of Eq. (14) is generally used because the H_1 ’s do not commute. Here the H_1 ’s take numerical values (see equations (12), (13), and (28)), and so we do not have here any commutation problems. Thus, the $L_{\text{bubble}}(k, t)$ from Eq. (11), for example, could have been written and substituted in Eq. (12) as

$$L_{\text{bubble}}(k, t) = i \int_{t_0}^t V_{klkl} e^{-i\epsilon_k(t_2-t_0)} G^-(l, t_1 - t_1) dt_1 \tag{15}$$

Note that for a general interaction, which is not of the bubble and open-oyster types and so not of the kind that may be discussed in terms of the static or dynamic Zeno effects (see the discussion after Eq. (24)), the H_1 from Eq. (14) do not assume numerical values and we have to take into account commutation relations. In this case the basic n times repeated interaction L^n will not assume the simple forms of Eq. (13) or (28) that lead directly to the sought-for Zeno results (see Eqs. (18)–(20) and (29)–(30)).

Now, we have to take into account all the possible orders of the n times repeated interaction process given by Eq. (13). For example, the second-order

²The diagrams that represent repetitions over improper self energy parts can be separated into unconnected self energy parts by removing the connecting particles lines. Generally, it is known (Mattuck, 1976) that such diagrams can not be summed over by using Dyson’s equation as we do below (see, for example, equations (18) and (34)), since these improper diagrams would have to be counted more than once (Mattuck, 1976).

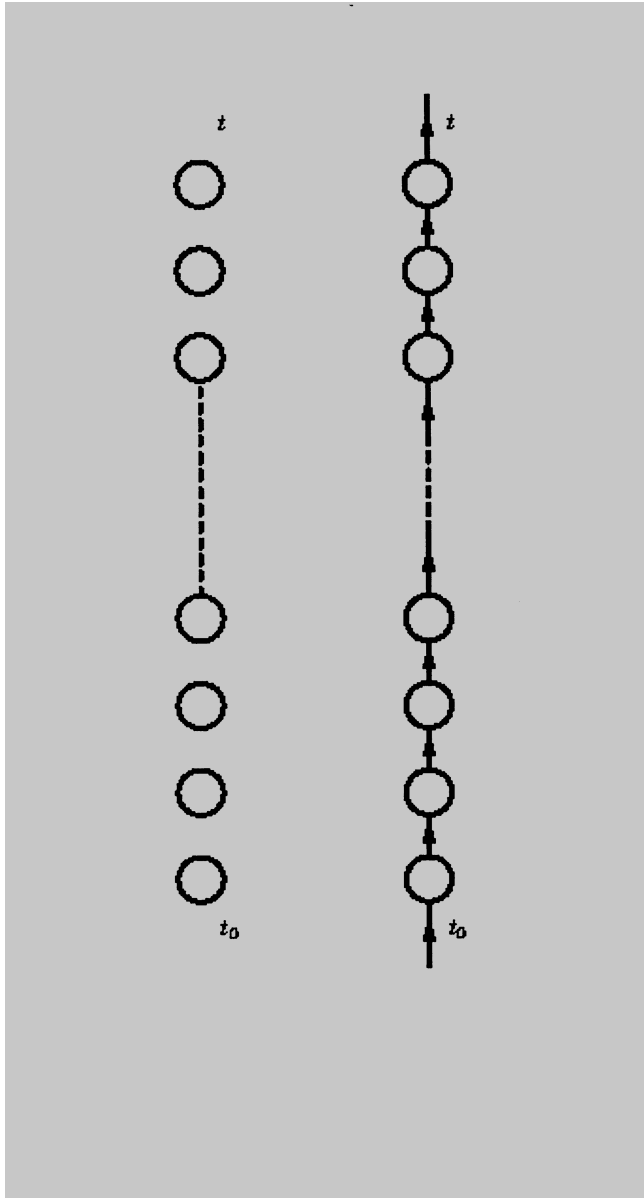


Fig. 1. The left hand side of the figure shows the n times repetitions of the bubble process which is represented as a circle. The right hand side shows these n times repetitions connected to each other by leading paths. The initial and final times are denoted on the graphs.

process, is

$$(L_{\text{bubble}}^n)^2(k, t) = (V_{klkl})^{2n} \sum_{m=n}^{+\infty} \sum_{p=n}^{+\infty} \frac{(t-t_0)^m}{m!(-i\epsilon_k)^{n-1-m}} \frac{(t-t_0)^p}{p!(-i\epsilon_k)^{n-1-p}} \quad (16)$$

and the n th order process

$$(L_{\text{bubble}}^n)^n(k, t) = (V_{klkl})^{n^2} \underbrace{\sum_{m=n}^{+\infty} \sum_{p=n}^{+\infty} \dots \sum_{q=n}^{+\infty}}_n \frac{(t-t_0)^{m+p+\dots+q}}{\underbrace{(m!p!\dots q!)}_n (-i\epsilon_k)^{n^2-n-(m+p+\dots+q)}}, \quad (17)$$

where the expression $(m + p + \dots + q)$ contains n terms. We want to demonstrate the Zeno effect in the dense measurement limit, that is, for very large n . So, repeating this n th order bubble interaction to all orders, taking the former equations into account, adding and subtracting 1, and using the Dyson's equation we obtain for the probability amplitude to find the system at time t in the same state it was at the initial time t_0 (compare with Eq. (5))

$$\begin{aligned} \lim_{n \rightarrow \infty} P_{\text{bubble}}^n(k, t) &= \lim_{n \rightarrow \infty} (L_{\text{bubble}}^{\text{free}} - 1 + 1 + L_{\text{bubble}}^n + (L_{\text{bubble}}^n)^2 \\ &\dots + (L_{\text{bubble}}^n)^n + \dots = \lim_{n \rightarrow \infty} \left(L_{\text{bubble}}^{\text{free}} - 1 + \frac{1}{1 - L_{\text{bubble}}^n} \right) = L_{\text{bubble}}^{\text{free}} \quad (18) \end{aligned}$$

The last outcome is obtained by using the last results of Equations (12) and (13) from which we obtain $\lim_{n \rightarrow \infty} L_{\text{bubble}}^n = 0$. $L_{\text{bubble}}^{\text{free}}$ is the probability amplitude to begin and end at the same state without any interaction. This no-interaction process, like the basic bubble interaction discussed here, is an n -times-repeated process. That is, $L_{\text{bubble}}^{\text{free}}$ is the n times repetitions of the free propagator given by Eq. (10), so that the time allocated for each is $\frac{(t-t_0)}{n}$. Thus, $L_{\text{bubble}}^{\text{free}}$, with the help of Eq. (10) and in the Zeno limit where $n \rightarrow \infty$, is

$$L_{\text{bubble}}^{\text{free}} = \lim_{n \rightarrow \infty} \left((-i) e^{-\frac{i\epsilon_k(t-t_0)}{n}} \right)^n = \lim_{n \rightarrow \infty} (-i)^n e^{-i\epsilon_k(t-t_0)} \quad (19)$$

From equations (18)–(19) we obtain for the Zeno limit of the probability of the bubble process

$$|L_{\text{bubble}}^{\text{free}}|^2 = 1 \quad (20)$$

That is, in the limit of the Zeno effect we obtain for the bubble process, when it is represented by either Eq. (1) (in the vacuum amplitude case) or by the more general Eq. (9), a probability of unity to begin and end in the same state.

We must again note that taking into account only the bubble process, from the large number of possible different processes, is the earlier Hartree method (Enz, 1992; Mahan, 1993; Mattuck, 1976) of dealing with the interacting many body system. But unlike this Hartree point of view in which the bubble interaction is

taken once to all orders, here in order to emphasize the important role of these identical repetitions to the Zeno effect this bubble interaction is taken n times to all orders where $n \rightarrow \infty$. Now, we discuss the other (excited) states of the system. The conventional procedure that yields the excited state energies is to find the poles of the propagator $G_{\text{bubble}}^+(k, w)$ (Mattuck, 1976) which is the Fourier transform of the propagator $G_{\text{bubble}}^+(k, t)$. The last propagator is the probability amplitude to find the system at the time t , *after interaction*, in the same state it has started from at the time t_0 , and it is, for the Zeno process, no other than the P_{bubble}^n we found in Eq. (18). Thus, we must transform this equation from the (k, t) representation to the (k, w) one. We do this by finding the (k, w) representation of $L_{\text{bubble}}^{\text{free}}$ from Eq. (19) using the Fourier transform method

$$\begin{aligned} L_{\text{bubble}}^{\text{free}}(k, w) &= \lim_{n \rightarrow \infty} \left((-i) \int_0^{+\infty} d\left(\frac{t-t_0}{n}\right) e^{-\frac{i\epsilon_k(t-t_0)}{n}} e^{\frac{iw(t-t_0)}{n}} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(\left(-\frac{e^{\frac{i(w-\epsilon_k+i\delta)(t-t_0)}{n}}}{(w-\epsilon_k+i\delta)} \right) \Big|_0^{+\infty} \right)^n = \lim_{n \rightarrow \infty} \left(\frac{1}{(w+i\delta)-\epsilon_k} \right)^n \end{aligned} \quad (21)$$

The δ in the exponent comes from multiplying by $e^{-\frac{\delta(t-t_0)}{n}}$, where δ is an infinitesimal satisfying $\delta \cdot \infty = \infty$, and $\delta \cdot c = 0$, (c is a constant) (Mattuck, 1976). We do this in order to remain with a finite result for this exponent when $(t-t_0) \rightarrow \infty$. The $L_{\text{bubble}}^{\text{free}}(k, w)$ is the n times repetitions of the free propagator $G_0^+(k, w)$ which is the (k, w) representation of $G_0^+(k, t_2-t_1)$ from Eq. (10). We are interested in the limit of very large n , and as seen from Eq. (21) when $n \rightarrow \infty$ we, actually, have a pole for each value of w that satisfies $|w-\epsilon_k| < 1$, that is, $\epsilon_k - 1 < w < \epsilon_k + 1$. There are no excited energies outside this range. We note that in the many body interaction picture the excited energy ϵ_k is equal to (Mattuck, 1976) the difference between the excited state energy of the interacting $N+1$ -particle system and the ground state of the interacting N -particle system. Thus, if the bubble process is performed once and the selective series of this once performed process is summed to all orders, as in the Hartree method, one obtains excited state energies at the value given by Eq. (23). But when this bubble process is repeated n times and the selective series of this n -times repeated process is summed to all orders, as we have just done in equation (12)–(18), we obtain from Eq. (21) excited state energies for all values of w that satisfy $|w-\epsilon_k| < 1$. That is, we obtain a large number (continuum) of extra excited energies that has been added *only because of these identical repetitions of the same bubble process*. This mechanism of obtaining physical results as a consequence of just repeating the same process which by itself, without these repetitions, does not yield these results has already been noted in (Aharonov and Vardi, 1980) in connection with rotations that occur only because of a large number of repetitions of the same measurement. Speaking in

terms of quasiparticles (Mattuck, 1976) we can write the (k, w) representation of $P_{\text{bubble}}^n(k, t)$ from Eq. (18), using Eq. (21), as

$$\lim_{n \rightarrow \infty} P_{\text{quasiparticle}}^n(k, w) = \left(\frac{1}{(w + i\delta) - \epsilon_k} \right)^n \tag{22}$$

$(\delta)^{-1}$ is the lifetime of the quasiparticle, and since δ is small $(\delta)^{-1}$ is very large, so these quasiparticles with the extra excited energies just mentioned have a very large lifetime. We must note that the relevant excited state energies w_{pole} obtained when the bubble process is performed once and the selective series of this once performed process is summed to all orders is just the Hartree w_{pole} (Mattuck, 1976).

$$w_{\text{pole}} = \epsilon_k + V_{klkl} - i\delta \tag{23}$$

When the bubble process is repeated n times, then as can be seen from equations (13), (18)–(19), and (21)–(22) the w_{pole} 's obtained do not depend on any potential V . This, as we have remarked, is in accord with the Aharonov–Vardi conclusion (Aharonov and Vardi, 1980) that the physical mechanisms that trigger the time evolutions of the system do not play an essential role, since the mere large number of repetitions of the same measurement is the cause of this time evolution. We note that Aharonov and Vardi show this for the spin $\frac{1}{2}$ particle example, but it is obvious from their representation that this conclusion is a general one. We have shown this for the bubble process for which a large number of repetitions results in excited energies that do not depend upon any potential. We show in the next section that if we have no repetitions then we do not have any excited energies.

In summary, we find that when the interaction involved does not end at the same state it has began from and if it is not repeated then no excited state results from such an interaction (see Eqs. (31)–(32) in the next section and the discussion that follows). If this interaction begins and ends at the same state as in the Hartree model then a single pole $w = \epsilon_k + V_{klkl}$ is found (see Eq. (23)). And when this interaction is repeated N times then in the limit of $N \rightarrow \infty$ one find a continuum of poles (cut) for all values of w that satisfy $|w - \epsilon_k| < 1$, where ϵ_k is the energy by which the involved system propagates during the interaction. That is, as has been remarked in (Aharonov and Vardi, 1980) the large number of repetitions produces new stable physical effects (see also Eq. (22) and the discussion that follows it) that do not appear in the absence of them. And the larger the number of these repetitions on the same time interval, as in the discussion here in which the repeated interaction is not taken by itself but by its N time repetitions where $N \rightarrow \infty$, the larger is the new stable physical effect as the cut found here (see Eqs. (21)–(22) and the relevant discussion there) instead of the single pole of the Hartree model. We note that the quasiparticles related to these poles have a very long lifetime so that once they are formed they do not decay fast.

3. THE ZENO EFFECT AND THE OPEN-OYSTER PROCESS

We, now, show that we can apply the Zeno effect (Aharonov and Vardi, 1980; Facchi *et al.*, 1999; Itano *et al.*, 1990; Misra and Sudarshan, 1997; Simonius, 1978) also for the general case, where the system ends at the time t in some specific state which is not identical to the initial one from which it has started at the time t_0 . In this context we do not use the standard Zeno effect at a state (where the system returns to the same state it has started from), as discussed in the previous section, but apply a Zeno effect along some definite Feynman path of possible states in the sense of Aharonov and Vardi (Aharonov and Vardi, 1980). That is, if we do dense measurement along any definite Feynman path of states then we make it actual in the sense that its probability amplitude is unity. Here we begin at some predetermined initial state and end at another predetermined final one. This aspect of the quantum Zeno effect in which the evolution of the relevant quantum system is *guided*, by means of dense measurement, to the corresponding prefixed final state is termed in (Facchi *et al.*, 1999) the dynamical quantum Zeno effect, in contrast to the usual quantum Zeno effect (in which the system starts and ends at the same state) which is termed in (Facchi *et al.*, 1999) the static quantum Zeno effect.

The propagator in this general case is the probability amplitude that if the system begins at the initial time t_0 in a specific state, then it will be found at another specific one at the later time t . As in the former section, in order to emphasize the important role of repetitions for the Zeno effect, the basic diagram is the n times repetitions of this interaction, where in the limit of dense measurement n becomes very large number. Thus, the terms of the infinite series representing the propagator signify the different orders of this n -repeated-interaction. In this case the repetitions is along some definite path connecting the initial and final states, and not local repetition as in the bubble example.

We choose, as in the bubble case, some example that may be described from two points of view. One is the situation when the interaction is triggered by an external potential that acts n , $2n$, $3n$ times etc. The other, more natural, interaction is that caused by the correlations between different particles that comprise the system. Unlike the bubble case, in both points of view there must be a connecting path between any two neighboring interactions since they are not identical to each other, as will be explained in detail later. Here the initial state of each such interaction is not identical to the initial state of the former one, *but to its final state*. The only difference between the external potential situation and the correlation-between-particles one is in the character of the interaction which in the former case is denoted by V_{kl} , that is, a particle that begins at state k is interacted upon by an external potential that changes its state to that of l (compare with the external potential situation of the bubble case in which a particle begins and ends at the same state, and therefore the external potential is denoted

by V_{kk}). In the correlation-between-particles situation this interaction is denoted by V_{lkk} (Mattuck, 1976; compare with the V_{klkl} of the correlation-between-particles situation of the bubble case).

A fundamental interaction in which the system ends at the time t in a state different from the one with which it has started from at the initial time t_0 is, for example, what is termed the open-oyster diagram (Mattuck, 1976). We must remark that this interaction is calculated to be (Mattuck, 1976) as one in which the particle that left the interaction site at the later time t is in the same state k with which another particle enters the interaction site at the initial time t_0 . Nevertheless, we discuss here another version of this interaction in which the particle that leaves the interaction site at the time t is in the state $l > k$, and not in the initial one k . We also call this interaction open-oyster. In the external potential version of this interaction an incoming particle at state k enters the potential region at the time t_0 . Then at time t the potential knocks another particle out of the state l_1 into state l , thus creating a particle in state l , and a hole in state l_1 . At the same time t the particle in k is knocked into the hole in l_1 , and thus annihilated with it. The particle in l continues propagating out of the potential region. This process is referred to as an exchange scattering (Mattuck, 1976), compared to the forward scattering of the bubble process in which the particle emerges in the same direction (i.e, momentum state) as it has entered. On the right hand side of Fig. 2 we see this open-oyster interaction, and on the left hand side of it we see n times repetitions of this process over the same time interval $(t - t_0)$. In the energy-time representation the probability amplitude for the occurrence of the open-oyster process is given by (Mahan, 1993; Mattuck, 1976):

$$L_{\text{open-oyster}}(k, t) = i \int_{t_0}^t V_{lk} G^-(l_1, t_1 - t_1) G_0^+(k, t_1 - t_0) G_0^+(l, t_2 - t_1) dt_1 \quad (24)$$

The difference between the bubble process that may represent the static Zeno effect (Aharonov and Vardi, 1980; Facchi *et al.*, 1999) (when repeated a large number of times), and the open-oyster process, that may be regarded as an example of the dynamic Zeno effect (Facchi *et al.*, 1999) (when performed many times), can be understood in the following way (Aharonov and Vardi, 1980; Facchi *et al.*, 1999): Suppose we have a family of states denoted as ϕ_k , where $k = 0, 1, 2, \dots, n$, such that $\phi_0 = \psi(0)$, where $\psi(0)$ is the initial state of the quantum system. We assume that successive states differ infinitesimally from one another, so that we have $\langle \phi_{k+1} | \phi_k \rangle \approx 1$. Denoting, as before, the total finite time of the n repeated interactions by $(t - t_0)$, and the time it takes to perform each such interaction by δt we have $\delta t = \frac{(t-t_0)}{n}$. Now, the open-oyster interaction may be regarded as, actually, projecting the evolving wave function at the time $t_k = k\delta t$ on the state ϕ_k . So when n becomes very large in the limit of the Zeno effect we obtain actually $\psi(t) = \phi_n$. This is the dynamic Zeno effect of (Aharonov and Vardi, 1980; Facchi *et al.*, 1999). The static Zeno effect is the special case when $\phi_k = \phi_0 = \psi(0)$ for all k .

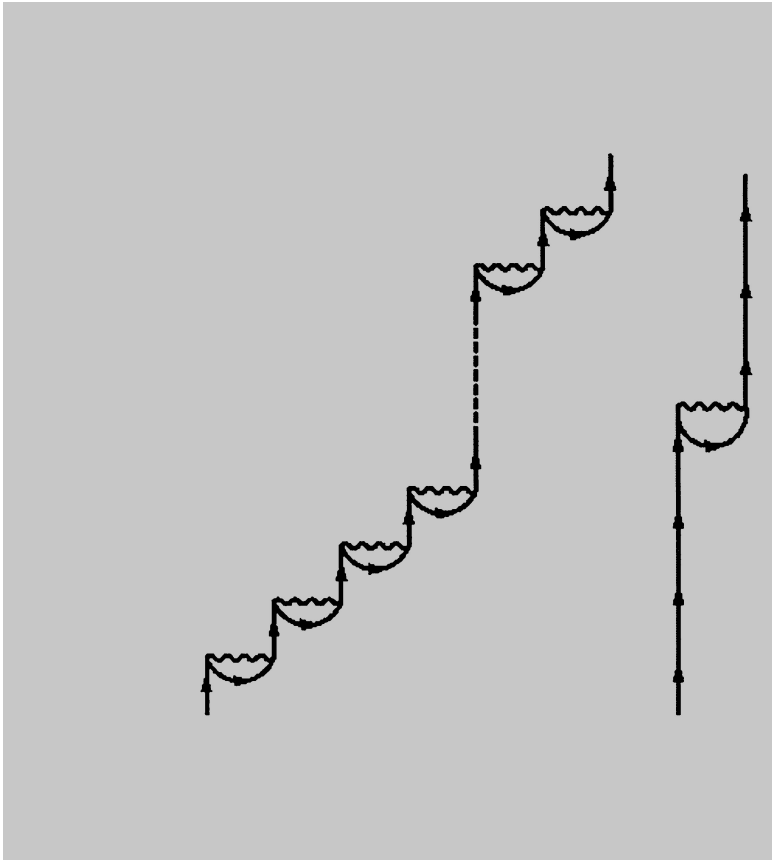


Fig. 2. The right hand side of the figure shows the fundamental open-oyster process, and the left hand side shows this process repeated n times over the same time interval.

If we describe this process in terms of the correlation between the different particles of the system then in this interaction an incoming particle in state k performs in a simultaneous manner several tasks; (1) it strikes another particle from state l_1 to state l , (2) creates a hole in l_1 , (3) is annihilated with the hole in l_1 , and the particle in l leaves the system. The open-oyster interaction is written now as

$$L_{\text{open-oyster}}(k, t) = i \int_{t_0}^t V_{kkl} G^-(l_1, t_1 - t_1) G_0^+(l_1, t_1 - t_0) G_0^+(l, t_2 - t_1) dt_1 \tag{25}$$

Now, since the last two equations (24) and (25) are identical to each other, except for the subscripts of the potential V , we concentrate our attention on Eq. (25) with

the understanding that what we say about it holds also for Eq. (24). V_{lkk_l} denotes the interaction just described, and the G_0^+ 's are the free propagators given by Eq. (10). We must note again that the successive repetitions of the open-oyster interaction, required for the discussion of the dynamic Zeno effect, are not characterized as being identical to each other, as in the bubble process, but that each such fundamental interaction begins from the point (state) in which the former interaction ends. Thus, we have to take into account the path that connects each two such neighbouring interactions. This connecting path is, of course, the free propagator $G_0^+(k, t - t_1)$. Substituting now from Eq. (10) into Eq. (25), and assuming that V_{lkk_l} does not depend on t we obtain

$$\begin{aligned}
 L_{\text{open-oyster}}(k, t) &= -i \int_{t_0}^t V_{lkk_l} G^-(l_1, t_1 - t_1) e^{-i\epsilon_k(t_1-t_0)} e^{-i\epsilon_l(t-t_1)} dt_1 \\
 &= V_{lkk_l} e^{-i(\epsilon_l t - \epsilon_k t_0)} \frac{e^{-i(\epsilon_k - \epsilon_l)t} - e^{-i(\epsilon_k - \epsilon_l)t_0}}{-i(\epsilon_k - \epsilon_l)} \\
 &= V_{lkk_l} \frac{e^{-i\epsilon_k(t-t_0)} - e^{-i\epsilon_l(t-t_0)}}{-i(\epsilon_k - \epsilon_l)} \tag{26}
 \end{aligned}$$

Where we have used the value of 1 for $-iG^-(l_1, t_1 - t_1)$. Using Eq. (25) we write for the n th order open-oyster process

$$\begin{aligned}
 L_{\text{open-oyster}}^n(k, t) &= \int_{t_0}^t \int_{t_0}^{t_1} \int_{t_0}^{t_2} \dots \int_{t_0}^{t_{n-1}} V_{k_1 k k k_1} V_{k_2 k_1 k_1 k_2} \dots V_{k_n k_{n-1} k_{n-1} k_n} \\
 &\cdot e^{-i\epsilon_k(t_1-t_0)} e^{-i\epsilon_{k_1}(t_2-t_1)} e^{-i\epsilon_{k_2}(t_3-t_2)} \dots e^{-i\epsilon_{k_{n-1}}(t_n-t_{n-1})} e^{-i\epsilon_{k_l}(t-t_n)} \\
 &\cdot dt_1 dt_2 dt_3 \dots dt_n = (V)^n e^{-i(\epsilon_{k_l} t - \epsilon_k t_0)} \int_{t_0}^t e^{-i(\epsilon_k - \epsilon_{k_l})t_1} dt_1 \\
 &\cdot \int_{t_0}^{t_1} e^{-i(\epsilon_{k_1} - \epsilon_{k_2})t_2} dt_2 \int_{t_0}^{t_2} e^{-i(\epsilon_{k_2} - \epsilon_{k_3})t_3} dt_3 \\
 &\dots \int_{t_0}^{t_{n-1}} e^{-i(\epsilon_{k_{n-1}} - \epsilon_{k_n})t_n} dt_n, \tag{27}
 \end{aligned}$$

where we have assumed that for large n all the potentials that transfer the system between two neighbouring states are equal to each other, that is, $V_{k_1 k k k_1} = V_{k_2 k_1 k_1 k_2} = \dots = V_{k_{n-1} k_{n-1} k_n} = V$. Carrying out the n integrals of the last equation we obtain an expression with 2^{n-1} terms, each of which is a fraction with a numerator that is a difference of exponentials in the energies ϵ_{k_i} 's multiplied by the times t_i , and the denominator is a multiplication of n different factors. This 2^{n-1} terms expression can be grouped into n different groups in which the number of terms are arranged as $1 + \sum_{i=0}^{(n-2)} 2^i$. All the terms of the same group have the same numerator up to a sign, but a different denominator, so we can reduce the number of all the terms of

each group to 1 by taking the common denominator of all the terms that belong to the same group. In such a way the total number of terms of the original expression is reduced from 2^{n-1} to n . Thus, we obtain

$$\begin{aligned}
 &L_{\text{open-oyster}}^n(k, t) \\
 &= (V)^n \sum_{m=0}^{(m=n-1)} \frac{(-)^l (e^{-i(\epsilon_k + \epsilon_{k_n} - \epsilon_{k_{n-m}})(t-t_0)} - e^{-i\epsilon_{k_n}(t-t_0)})}{(-i)^n \prod_{i=0}^{(i=n-(m+1))} (\epsilon_{k_i} - \epsilon_{k_{n-m}}) \prod_{(i=n-m)}^{(i=n-1)} (\epsilon_{k_{n-m}} - \epsilon_{k_{i+1}})}
 \end{aligned} \tag{28}$$

It can be seen that all the n numerators of the last equation are differences of sines and cosines, whereas each one of the corresponding n denominators is a product of n factors that are differences of energies. When n is very large, which we always assume in this work, we have $\epsilon_{k_i} \approx \epsilon_{k_{i+1}}$ (since neighbouring states differ infinitesimally), so in this limit we have at least two factors in each denominator that tend to zero. Thus, although all the n terms of Equation (28) are multiplied by the factor V^n (V is a probability amplitude that satisfies $0 \leq V \leq 1$) we obviously have $\lim_{n \rightarrow \infty} L_{\text{open-oyster}}^n = \infty$.

We are interested, as in the bubble case, in the repetitions to all orders of $L_{\text{open-oyster}}^n$ from Eq. (28). Beginning from this equation it is not hard to obtain the various orders of $L_{\text{open-oyster}}^n$. So, if we take the infinite series (that denotes the various orders of the n repetitions process $L_{\text{open-oyster}}^n$), adding and subtracting 1, and taking the relation $\lim_{n \rightarrow \infty} L_{\text{open-oyster}}^n = \infty$ into account we obtain, using the Dyson's equation, for the general probability amplitude in the Zeno limit

$$\begin{aligned}
 \lim_{n \rightarrow \infty} P_{\text{open-oyster}}^n(k, t) &= \lim_{n \rightarrow \infty} (L_{\text{open-oyster}}^{\text{free}} + 1 - 1 + L_{\text{open-oyster}}^n \\
 &+ L_{\text{open-oyster}}^{2n} + \dots) = \lim_{n \rightarrow \infty} \left(L_{\text{open-oyster}}^{\text{free}} - 1 \right. \\
 &\left. + \frac{1}{1 + L_{\text{open-oyster}}^n} \right) = \lim_{n \rightarrow \infty} L_{\text{open-oyster}}^{\text{free}} - 1 \tag{29}
 \end{aligned}$$

$L_{\text{open-oyster}}^{\text{free}}$ is the amplitude for our system to begin in some specific initial state ϕ_k at the time t_0 , and end in another different state ϕ_l at the time t without any interaction whatever on our system. This no-interaction process is obviously zero if the final state is different from the initial one (see, for example, Mahan, 1993; Mattuck, 1976), so we obtain for the *probability* of the open-oyster process in the Zeno limit

$$\lim_{n \rightarrow \infty} |P_{\text{open-oyster}}^n|^2 = 1 \tag{30}$$

Thus, we see that in this limit we obtain for the open-oyster process a probability of unity to end at a specific prescribed state different from another specific initial one.

We now show that we have no excited state energies for the open-oyster process in the Zeno limit. For this purpose we must find, in this limit, the poles of the propagator $P_{\text{open-oyster}}(k, w)$ which is the Fourier transform of the propagator $P_{\text{open-oyster}}(k, t)$ given by Eq. (29). Thus, using the Fourier transform procedure, multiplying by $e^{-\delta(t-t_0)}$ (Mattuck, 1976), and using $\lim_{n \rightarrow \infty} L_{\text{open-oyster}}^{\text{free}} = 0$ we obtain

$$\lim_{n \rightarrow \infty} P_{\text{open-oyster}}^n(k, w) = - \int_0^\infty d(t - t_0) e^{i(w+i\delta)(t-t_0)} = \frac{1}{w + i\delta}, \quad (31)$$

where the δ is, as in Eq. (21) (see the discussion after Eq. (21)), an infinitesimal quantity that satisfies $\delta \cdot \infty = \infty$, and $\delta \cdot c = 0$, where c is some finite number. This δ has been introduced in order to have a finite result for the exponent of Eq. (31) in the limit $(t - t_0) \rightarrow \infty$ (see Appendix I in (Mattuck, 1976)). From the last equation we obtain that the poles of $\lim_{n \rightarrow \infty} P_{\text{open-oyster}}^n(k, w)$, which are the excited energy states of the physical system are

$$w_{\text{pole}}^{\text{open-oyster}} = 0 \quad (32)$$

That is, there exists no excited energy states in the Zeno limit of the open-oyster process. The reason, as we have remarked, is the absence of local repetitions in the version we have adopted here for the open-oyster process. That is, we discuss here a process in which the state of the particle that leaves the system is different from the state of the one that enters. And when this process is repeated the initial state of the entering particle in the repeated process is the final state of the leaving particle in the former one. Thus, this process is not locally repeated, and this absence of repetitions entails the absence of excited states for the system. That is, all the energies of the $N + 1$ -particle system are equal, in the Zeno limit, to each other and to the ground state energy of the N -particle system (see Mattuck, 1976, p. 41). In contrast to this situation, when we have local repetitions of some process, then we have excited states of the physical system. That is, if the selective series of this process is composed of repeated to all order terms like the Hartree selective series of the bubble process, then excited states are obtained (see Eq. (23)). Many more additional excited states are obtained when this summation to all orders is over the n -times repetitions of this process as we have obtained for the bubble process in the former section (see Eq. (21)). Now, if we discuss this open-oyster process from the conventional point of view (Mattuck, 1976) where the energy of the leaving particle is the same as that of the entering one, and the summation to all orders is over the once-performed open-oyster process and not over the n th times repetitions of it, then we obtain for the w_{pole} (Mattuck, 1976)

$$w_{\text{pole}} = \epsilon_k + V_{lkk} - i\delta, \quad (33)$$

where V_{lkk} is the physical interaction that generates this open-oyster interaction. That is, the excited state energies of the system are determined by these repetitions,

as has been remarked in (Aharonov and Vardi, 1980) (see the discussion after Eq. (21))

We must note that the result of Eq. (30) is obtained not only for the open-oyster case, but also for any other arbitrary interaction for which the amplitude M to ends in a specific state different from the initial one satisfies $M > 1$. If we denote the propagator (the full propagator, not the free one) of such interaction by P_{Zeno} , its free propagator by P_{free} , and adding and subtracting 1 the propagator takes the following form

$$\begin{aligned} \lim_{n \rightarrow \infty} P_{\text{Zeno}} &= \lim_{n \rightarrow \infty} (P_{\text{free}} - 1 + 1 + M^n + M^{2n} + M^{3n} + \dots) \\ &= \lim_{n \rightarrow \infty} \left(P_{\text{free}} - 1 + \frac{1}{1 - M^n} \right) = -1 \end{aligned} \quad (34)$$

In obtaining the result of Eq. (34) we made use of the facts that $P_{\text{free}} = 0$, and $M > 1$ so that $\lim_{n \rightarrow \infty} M^n = \infty$. We see, therefore, that also for the general case, where the system reaches at the time t a different state from that in which it started, we get a probability of 1 in the dense measurement limit. Thus, we see that the Zeno effect (Aharonov and Vardi, 1980; Facchi *et al.*, 1999, Itano *et al.*, 1990; Misra and Sudarshan, 1977; Simonius, 1978) may be effective in the framework of quantum field theory.

4. CONCLUDING REMARKS

We show that the Zeno effect may be discussed also in the context of quantum field theory. We have used in Section 2 the Dyson's equation and the bubble example to demonstrate the static Zeno effect, in which the initial and final states of the system are the same. In Section 3 we have used the open-oyster example and the Dyson's equation to demonstrate the dynamic Zeno effect, in which the initial and final states of the system are different. In this work the Dyson's equation has been used to infinitely sum to all orders over the n times repetitions of these two processes. It has been shown in Sections 2 and 3 that the probability amplitudes to find the final state of the system identical to the initial one in the bubble case, or different in the open oyster case tend both to unity as the number of repetitions n becomes large.

We have found in Section 2 that repeating the bubble process a large number of times in a finite total time results in obtaining a large number (cut) of additional excited energy states that emerge only because of these repetitions. By this we have corroborated the same conclusion arrived to by Aharonov and Vardi with respect to spin rotation. We have found, accordingly, in Section 3 for the open-oyster process that the absence of any repetition results in the absence of excited state energies.

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